OLD PROBLEM–NEW PROGRESS

How a Century-old Obstacle Has Been Avoided in Dealing with the Astronomer's "Problem of Three Bodies"... Special Problem Just Solved

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ASTRONOMICAL calculations are a literary byword—everyone knows that they are of vast intricacy. A good many of us have heard that some of the worst of them deal with mysterious things called "perturbations"—but fewer of us could tell much more about them.

The trouble with the astronomers is that they can make their observations with great accuracy (which has itself become a byword); and they naturally want to predict the motions of the bodies which they observe at least as accurately as they can measure them.

This would be no trouble at all if we had to deal with only two bodies at a time—such as a planet going around the sun. Under the sun's gravitational attraction, it would pursue a fixed elliptical orbit, returning to the same place at exactly equal intervals. If the orbit were a circle, it would move uniformly in it, and prediction would be child's play. The more eccentric the orbit, the more uneven the motion; but the position at any time may still be calculated by formulas workable by anyone who knows a bit of trigonometry.

With three bodies present—say Jupiter, Saturn, and the sun—the problem becomes extraordinarily complex. More than a century of study by the most competent mathematicians has made it certain that, if any general formula could be obtained to take care of the case of any three bodies (whatever their masses and motions) it would be so intricate that it would be of no practical use at all for numerical calculation. Fortunately for the astronomers, they have to deal with cases of special types, in which solutions can be obtained by a succession of approximations.

The handsome exterior of the Optical Shop on the campus at the California Institute of Technology, in Pasadena, within which the 200-inch mirror disk is now being ground—as is shown on the preceding page

THE planets, for example, are so small compared with the sun that the attraction of one upon another is never as great as a hundredth part of the sun's attraction on it, and usually very much less. We start, then, by treating the motion of Saturn as if Jupiter were not there, and vice versa. This gives formulas for their motions (each in a fixed orbit) from which their positions can be approximately, though not exactly, calculated. Using these computed positions, we may now calculate the attraction of each upon the other, and then find the changes which their attractions will produce in their motions. These changes are the "perturbations." To compute them is by no means a simple problem, but one that has been solved. Armed with this information, and correcting our original calculations of the planets' positions at a given moment for the perturbations, we can use these corrected positions to get improved values of the perturbing forces. These again lead—after a great amount of labor—to more precise values of the perturbations themselves. A third similar step is necessary only for the most massive planets, Jupiter and Saturn.

The results of these vast calculations may be described rather simply. We may suppose the orbit of each planet to remain an ellipse, but very slowly to change its shape and shift its position in space—though its size and the period of revolution remain unaltered. The actual planet will never get very far from an imaginary one, which moves in this "mean ellipse" in accordance with Kepler's simple laws, though it may be nearer the sun, or farther off, ahead of the fictitious planet or behind it, above the orbit plane or below it. The amount of these deviations may be represented by the sum of a great number of "terms," each of which has a fixed range, or amplitude, and a fixed period. A single such term may be described geometrically by uniform motion in a small circle exactly like the famous epicycles by means of which Ptolemy, 1800 years ago, sought to calculate the planets' motions.

The modern "tables of the planets" indeed are a glorified collection of epicycles, far more numerous than Ptolemy's. What gravitational theory does is to enable us to calculate accurately the period and radius of each epicycle.

Theoretically, the number of these terms or epicycles is infinite; but in practice only a moderate number are large enough to have to be considered, even in the most refined work, so that the "planetary theory" is a practical success.

The lunar theory, which deals with the motion of a satellite under the attractions of its primary and the sun, differs widely in its analytical details, but ends with formulas of the same type—a slowly changing mean orbit, and a host of epicycles or periodic terms. Like the planetary theory, it succeeds in practice, though Brown has had to include almost 700 terms in his expression for the moon's longitude. But this success depends on the fact that the orbits of the planets, and of the moon, are roughly circular—having small eccentricities—and are inclined to one another by but a few degrees. The coefficient of each term (in plain words, its size) comes out of the analysis in the form of a power-series in the eccentricities and inclinations. For most of the infinite number of terms the coefficients involve such high powers of these small quantities that they are negligibly small. When precise perturbations are calculated for Pluto, with its high eccentricity and inclination, the number of "sensible" terms will probably be very large. (It will be many years before our knowledge of its orbit will be accurate enough to justify this heavy labor.)

To apply the same methods to a comet would probably be useless—it is very doubtful, to say the least, whether the series would be convergent. Fortunately, there are other, and much easier, methods for computing, as accurately as we like, the perturbations of a comet by the planets—step by step, and month by month, for as long as our patience holds out—and this suffices in practice, since no comet has yet been observed for as many as 40 returns, and only two for more than 20.

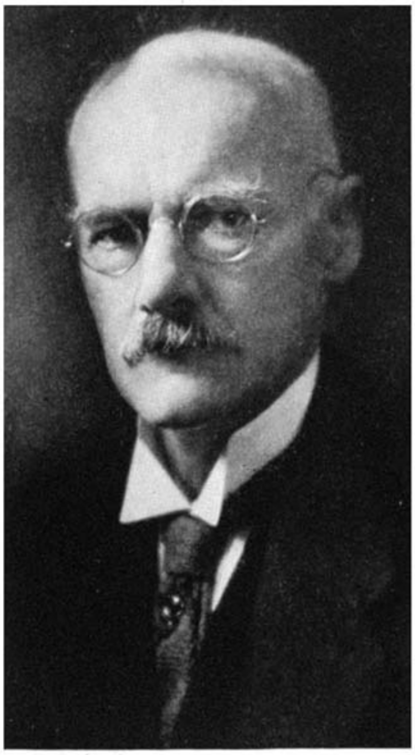
But there are other cases, of much theoretical interest, in which the step-by-step process would not work.

FOR a considerable percentage of the double stars, for which orbits have been computed, one of the components has been found to be a spectroscopic binary—a far closer double. For example, Castor consists of two close pairs, with periods of three and nine days, revolving about one another in some 300 years.

In such systems, the smaller orbit is often, and the larger one almost always, highly eccentric, so that the "classical" methods already described are not available. It is also impracticable to calculate the motion in the close pair, step by step, over thousands of revolutions.

Here is a new problem presented by nature, demanding a new method of mathematical attack, and the problem has just been solved, in a brilliant fashion, by the most experienced veteran in the field of celestial mechanics —E. W. Brown. Professor Brown starts from scratch—with the general equations of motion—but applies a new method of approximation. In all known cases of this sort the period of the close pair is but a small fraction of that of the wide pair. Call this fraction m. A general solution of the problem would involve terms containing m, m^2, m^3, and so on. A first approximation, and one quite adequate for the present purpose, may be obtained by neglecting the very small terms involving m^2, and so on. When this is done, he finds that the mathematical analysis is simplified to an unexpected and remarkable degree. It is no longer necessary to expand in powers of the eccentricities and inclinations; instead, general formulas are obtained, which hold good even for very large values of the eccentricities of both the large and small orbits, and of their mutual inclination.

Nothing like this has ever been met with before in the century and a half of past investigation. It is indeed a new case in celestial mechanics, and the most notable theoretical advance in many years.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Professor E. W. Brown of Yale, a specialist in celestial mechanics

The motion of the wide pair is not perceptibly affected by the duplicity of the close pair; but the orbit of the latter may change considerably. During each revolution of the close pair there are departures from simple elliptic motion—usually too small to be important. During each revolution of the wide pair the orbit of the close pair, and the position of the stars in this orbit, undergo periodic changes. Finally, there are slow changes. The point of closest approach of the stars (periastron) moves forward, the node of the orbit-plane on that of the larger orbit retrogrades, and both the eccentricity and inclination fluctuate by large amounts. These changes are themselves periodic, and require for their completion as many (or more) revolutions of the wide pair as there are revolutions of the close pair in one of the wide. These changes are illustrated by Professor Brown's calculations for the system of *Xi Ursae Majoris*—which has already been described in these columns. [June, 1936, page 315. —*Ed.*]

The wide pair with a period of 60 years has an eccentricity of 0.41 and an inclination of 57 degrees to the "plane of the sky" on which it appears to be projected. It has been well observed over nearly two revolutions. The brighter component is a spectroscopic binary with a period of 1.83 years and an eccentricity of 0.53. Precise photographic measures of the apparent separation of the wide pair show an oscillation which indicates that the smaller orbit is inclined 85 degrees to the "sky" and 38 degrees to the other orbit. The companion is too faint to see separately.

The other component of the wide pair is also double, with a period of four days; but this pair is so close that for all practical purposes it may be treated as a single body. The first pair, however, affords an almost ideal case for the application of the new theories. The periodic changes in the smaller orbit, which repeat themselves after 60 years, alter the eccentricity by a total range of 0.03, the inclination by 3 degrees, and the longitude in the orbit by a little over 1 degree. Could we observe the pair from close by, these changes would be conspicuous; but at its actual distance these effects are at the very limit of observation, if not beyond it.

THE gradual changes are much greater, but very slow. The forward revolution of the periastron takes 24,000 years to complete a circuit, the retrogression of the node, 4600 years. The eccentricity ranges from 0.54 to 0.19, and the mutual inclination of the orbits from 37 to 47 degrees. These changes repeat themselves after 1900 years.

Could we observe the pair long enough, we would therefore find easily observable changes. The orbit plane, in particular, may at times have an inclination—from our viewpoint—of 57–47 degrees, or only 10 degrees. At other times it may be exactly edgewise toward us, and it is possible that one star may eclipse the other. Unfortunately, this will not happen for many centuries.

Of all the known cases, this is the one in which the short period is the largest fraction of the longer one, and hence the one in which the perturbations are the greatest. For such a star as Castor, the periodic changes in the short period orbits must be far too small, and the progressive changes far too slow, to be detected by observation. In the triple star *Zeta Cancri*, a pair with a 60-year period has a slowly moving distant companion which must require several centuries for its revolution. There are several other such stars; and, when they have been followed nearly around their orbits, there will be a fine chance for the application of the present theory—which will then be some centuries away from being new. Until then, the problem may be regarded as closed. —*Princeton, N. J., June 2, 1937.*